Impulsive Noise Suppression by use Robust Linear Combinations of Order Statistics

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Abstract. In this paper, we present a new nonlinear filter to suppress the impulsive noise in image processing applications. The proposed filter uses the linear combinations of order statistics in its filtering scheme and to improve the impulsive noise suppression capabilities we introduce in the proposed filtering approach the Rank M-type (RM) estimator. We use the simple influence function and different distribution functions to calculate the weighted coefficients of proposed filter. Extensive simulation results demonstrate that the proposed filter outperforms the balancing between noise suppression and detail preservation in comparison with other variants of linear combinations of order statistics based filters.

1 Introduction

Many different classes of filters have been proposed for removing noise from images [1, 2]. They are classified into several categories depending on specific applications. Linear filters are efficient for Gaussian noise removal but often distort edges and have poor performance against impulsive noise [1, 2]. Nonlinear filters are designed to suppress noise of different nature, they can remove impulsive noise and guarantee detail preservation. They have proven to be exceptionally useful in many image restoration applications. Because of their robust properties, some of these filters have been used when the images are corrupted by non-Gaussian noise [1, 2].

One of the best known nonlinear filter classes is based on the order statistics [3]. It uses the concept of data ordering. There is now a plethora of nonlinear filters based on data ordering [4-7]. Among them are the L-filters whose output is defined as a linear combination of the order statistics [1-7]. Some examples of L-filters are the *Combination* (C-L) filter [5] and the *normalized least mean squares L* (NLMS-L) filter [6].

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Recently, we presented the Rank M-type K-Nearest Neighbor (RM-KNN) filters [8, 9] for the removal of impulsive noise in image processing applications. These filters are based in the combination of the KNN filter [2] and the RM-estimator [8, 9]. The use of KNN algorithm provides a good detail preservation. The RM-estimator utilizes the combination of the *R*-estimator and the *M*-estimator with different influence functions to improve the noise supresion and detail preservation [9].

In this paper, we present a new class of *L*-filter. The proposed filtering scheme uses the RM-estimator into the *L*-filter according with the RM-KNN filtering approach [9]. The use of the RM-estimator with the simple cut influence function [1-3, 9] in the *L*-filter improves the properties of noise suppression and detail preservation in comparison with other classes of *L*-filter. We also introduce the use of an impulsive noise detector [10] to improve the properties of noise suppression and detail preservation in the proposed filtering scheme. Additionally, we use an exponential, gaussian, and laplacian distribution functions [7] to calculate the coefficients of the new *L*-filter. Extensive simulations in different images with different impulsive noise percentages were realized. The criteria used to compare the restoration performance of various filters were the *peak signal-to-noise ratio* (PSNR) for the evaluation of noise suppression, the *mean absolute error* (MAE) for quantification of edges and fine detail preservation [1, 2].

2 Proposed Filtering Scheme

Recently, we proposed the combined Rank M-type (RM) estimator to improve the capability of KNN filter. It was demonstrated that the RM-estimator provides good properties of impulsive noise suppression and detail preservation [9]

$$\theta_{\mathrm{RM}} = \mathrm{MED}\left\{X_i \tilde{\psi}\left(X_i - \mathrm{MED}\{\vec{\mathrm{X}}\}\right), i = 1,\dots,N\right\} \qquad (1)$$

where $\tilde{\psi}$ is the normalized function ψ: $\psi(X) = X\tilde{\psi}(X)$, X_i are data samples, and $MED(X)$ is the median of data samples.

In this paper, we proposed to use the RM-estimator (1) in the filtering scheme of the *L*-filter to improve its properties of noise suppression and detail preservation. The *L*-filter is given by [1-3, 5-7]:

$$\theta_L = \sum_{i=1}^{n} a_i \cdot X_{(i)} \qquad (2)$$

where $X_{(i)}$, $i = 1,\dots,n$ are the ordered data samples and a_i, $i = 1,\dots,n$ are the weighted coefficients of filter whose are calculated in the following form [1, 2]

$$a_i = \frac{\int_{\frac{i-1}{n}}^{\frac{i}{n}} h(\lambda)d\lambda}{\int_0^1 h(\lambda)d\lambda} \qquad (3)$$

where $h(\lambda)$ is a probability density function $f(x)$.

To introduce the RM-estimator in the scheme of *L*-filter, we should be to present the ordered data samples of *L*-filter as function of an influence function. For this reason, the *L*-filter is writing as:

$$\theta_L = \sum_{i=1}^{n} a_i \cdot \psi(x_i) \cdot x_i \qquad (4)$$

and

$$\psi(x_i) = \begin{cases} 1 & i \le (2L+1)^2 \\ 0 & \text{otherwise} \end{cases} \qquad (5)$$

where $(2L+1)^2$ is the filtering window size, $\psi(x_i)$ is the influence function used in the *L*-filter and $\psi(x_i) \cdot x_i$ are the ordered data samples according with the eq. (2).

Then, the new filter can be obtained by the combination of *L*-filter (4) and the RM-estimator (1), it can be writing as

$$\theta_{RM\text{-}L} = \frac{\mathrm{MED}\{a_i \cdot [x_i \cdot \psi(x_i - \mathrm{MED}\{\vec{X}\})], i = 1,\ldots,n\}}{a_{\mathrm{MED}}} \qquad (6)$$

where $\theta_{RM\text{-}L}$ is the output of the proposed filter, $x_i \cdot \psi(x_i - MED\{\vec{X}\})$ are the selected pixels in accordance with the influence function in a sliding filter window, a_i are the weighted coefficients used into the *L*-filter, and a_{MED} is the coefficient of *L*-filter used to obtain the median.

To improve the properties of impulsive noise suppression of the proposed filter we introduced an impulsive detector, this detector chooses that pixel is or not filtered. The impulsive detector used here is defined as [10]:

$$[(rank(X_{ij}) \le s) \vee (rank(X_{ij}) \ge N - s)] \wedge \left(\left| X_{ij} - MED(\vec{X}) \right| \ge U_2 \right) \qquad (7)$$

where X_{ij} is a pixel of interest (the central pixel in the filtering window), $s>0$ is a threshold, N is the length of the variation series, $MED(X)$ is the median of pixel into the filtering window, and $U_2 \ge 0$ is a threshold.

We also utilize the simple influence function in the proposed filtering approach. This function is given by [1-3, 8, 9],

$$\psi(x_i - \mathrm{MED}\{\vec{X}\}) = \begin{cases} 1, & \left|x_i - \mathrm{MED}\{\vec{X}\}\right| \le r \\ 0, & \left|x_i - \mathrm{MED}\{\vec{X}\}\right| > r \end{cases} \quad (8)$$

where r is a proposed parameter between 0-255 and $x_i - MED\{\vec{X}\}$ is the i-pixel minus the median of the pixels in the filtering window.

The weighted coefficients of the new type of L-filter were found using the following distribution functions [7],
the exponential distribution function [3, 7],

$$f(x) = \tfrac{1}{2} e^{-|x|} \quad (9)$$

and the laplacian distribution function [7],

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-m)^2}{\sigma^2}} \quad (10)$$

We note that the coefficients are calculated by each sliding filter window due that the influence function selects whose pixels are used and then compute the weighted coefficients of L-filter according with the number of pixels used into the filtering window.

3 Simulation Results

We obtained from the simulation experiments the properties of the proposed filter and compared it with other variants of L-filters proposed in the literature.

The criterion used to compare performance of noise suppression of filters was the *peak signal-to-noise ratio* (PSNR) [1, 2]

$$\mathrm{PSNR} = 10 \cdot \log\left[\frac{(255)^2}{\mathrm{MSE}}\right], \mathrm{dB} \quad (11)$$

and the *mean absolute error* (MAE) for evaluation of fine detail preservation [1, 2]

$$\mathrm{MAE} = \frac{1}{M_0 N_0} \sum_{i=0}^{M_0-1} \sum_{j=0}^{N_0-1} \left| e(i,j) - \hat{e}(i,j) \right| \quad (12)$$

where $\mathrm{MSE} = \frac{1}{M_0 N_0} \sum_{i=0}^{M_0-1} \sum_{j=0}^{N_0-1} [e(i,j) - \hat{e}(i,j)]^2$ is the *mean square error*, $e(i,j)$ is the original image; $\hat{e}(i,j)$ is the restored image; and $M_0 x N_0$ is an image size.

In our experiments, a 3x3 filter window (i. e. $m,n=-1,\dots,1$ and $(2L+1)^2=9$) is applied.

To determine the impulsive noise suppression properties of various filters the 256x256 standard test grayscale image "Lena" was corrupted by impulsive noise with an occurrence rate of 5 and 20%. Therefore, we present the simulation results when this image is corrupted with 0.05 and 0.1 of variance of multiplicative noise.

Table 1 shows the performance results in terms of PSNR in dB and MAE for the image "Lena" degraded with 5% and 20% of impulsive noise, and 0.05 and 0.1 of variance of multiplicative noise by use the proposed filter with different distribution functions and impulse detector. In the results of Table 1 we use the *L*-filter [7] to compare our approach due it was compared with other variants of *L*-filters and it demonstrated better properties of noise suppression and detail preservation. From this table one can see that the proposed filter provides better impulsive and multiplicative noise and detail preservation in comparison with the *L*-filter [7].

Table 1. Performance results in the image "Lena" obtained by the use of different filters

Filters	Impulsive noise				Multiplicative noise			
	5%		20%		0.05		0.1	
	PSNR	MAE	PSNR	MAE	PSNR	MAE	PSNR	MAE
Proposed filter without detector, exponential	25.99	7.97	23.65	10.86	20.30	19.14	18.45	24.02
Proposed filter without detector, laplacian	27.08	7.47	24.79	9.11	22.89	14.00	21.14	17.19
Proposed filter without detector, gaussian	28.05	6.12	25.59	7.38	24.64	10.89	22.84	13.48
Proposed filter with detector, exponential	27.10	7.00	24.64	8.79	21.19	16.98	19.24	21.62
Proposed filter with detector, laplacian	28.06	6.36	24.97	8.05	23.02	13.63	21.16	17.04
Proposed filter with detector, gaussian	28.73	5.64	25.47	7.02	24.54	11.01	22.68	13.71
L-filter [7]	25.25	8.03	23.01	10.24	23.48	11.79	22.07	14.20

Figure 1 presents the visual results in the image "Lena" according with the Table 1. We observe in this figure that the proposed filter shows better results in noise suppression and detail preservation in comparison with the *L*-filter [7].

The use of the proposed filter in image processing applications provides good results in terms of PSNR and MAE performances. To demonstrate the performances of the proposed filtering scheme we applied it for filtering of the SAR images, which naturally have speckle noise. The results of such a filtering are presented in the Figure 2 for the image "Pentagon". It is possible to see analyzing the filtering images that speckle noise can be efficiently suppressed, while the sharpness and fine feature are preserved using the proposed filter in comparison with the *L*-filter [7].

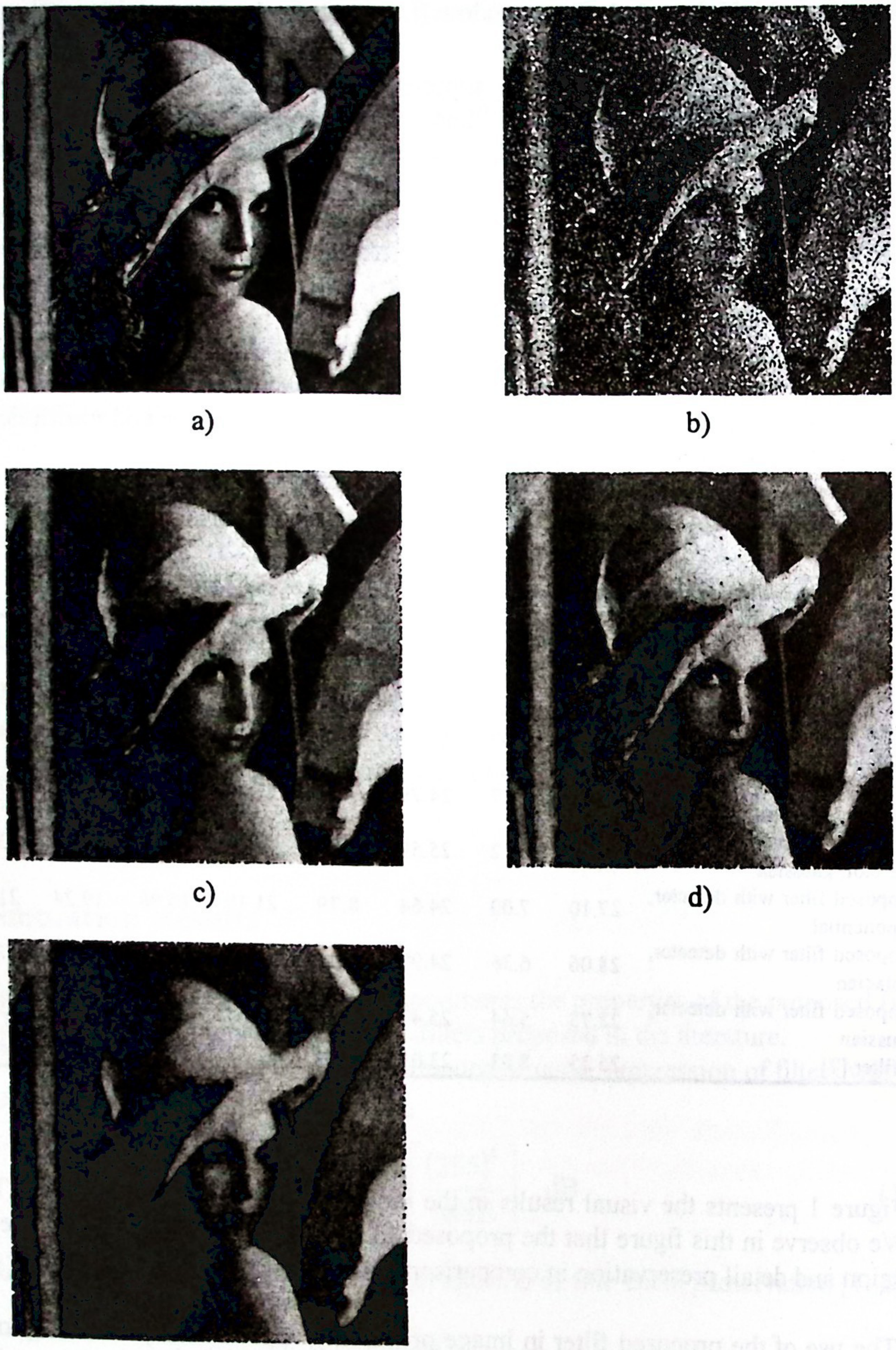
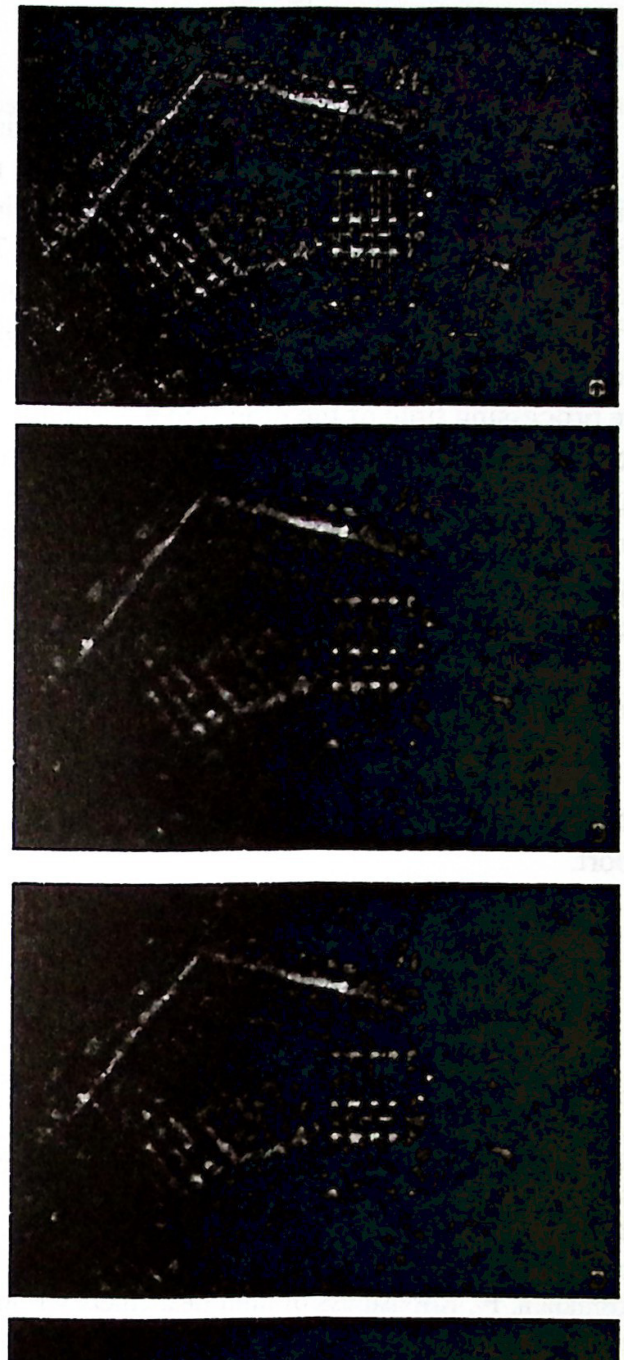


Figure 1. Visual results in the image "Lena", a) Original image "Lena", b) Degraded image with 20% of impulsive noise, c) Restored image with the proposed filter without detector, gaussian, d) Restored image with the proposed filter with detector, Gaussian, e) Restored image with the *L*-filter [7].



a) b) c) d)

Figure 2. Comparative results of despeckled SAR image. a) Original image "Pentagon", resolution 1m, source Sandia National Lab., b) Despeckled image with the proposed filter without detector, gaussian, c) Despeckled image with the proposed filter with detector, gaussian, d) Despeckled image with the *L*-filter [7].

4 Conclusions

We present a new class of L-filter that uses the robust RM-estimator in its filtering scheme for image processing applications. Extensive simulation results demonstrated that the proposed filter provides better impulsive noise suppression and detail preservation in comparison with the L-filter proposed in the reference [7].

The RM-estimator improves the performances in noise suppression and detail preservation in comparison with the traditional L-filter. The noise detector also improve the properties of noise suppression and detail preservation of the proposed filter and reduce the processing time of the proposed algorithm.

We should be to improve the properties of proposed filter to suppress better the speckle (multiplicative) noise in images. For this reason, we are working to improve the capability of proposed filter by means of use other size of the filter window, the coefficients of proposed filter provides good noise suppression and we will compute them by the use of other distribution function, and finally, the influence function provides sufficiently robustness to suppress the noise and we will probe with other influence functions proposed in the literature.

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